Rational Sentiments and Financial Frictions

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Models with financial frictions: shortcomings

Large macro literature featuring financial frictions

Bernanke and Gertler [1989]; Shleifer and Vishny [1992]; Kiyotaki and Moore [1997]; Bernanke et al. [1999]; Gertler and Kiyotaki [2010]; Gertler and Karadi [2011]; Bianchi [2011]; Mendoza [2010]; Brunnermeier and Sannikov [2014]; Brunnermeier and Sannikov [2015]; Phelan [2016]; Drechsler et al. [2018]; Moreira and Savov [2017]; Klimenko et al. [2017]; Bianchi and Mendoza [2018]; He and Krishnamurthy [2019].

Problem 1: reproducing the severity and suddenness of financial crises

$\Rightarrow$ Add systemic bank runs

Gertler and Kiyotaki [2015]; Gertler et al. [2020]; Mendo [2020]

Problem 2: generate booms that are prone to bust

$\Rightarrow$ Add non-rational beliefs

Krishnamurthy and Li [2020]; Maxted [2020]

This paper:

- this class of economies has unstudied equilibria (sunspot equilibria)
- sunspots help alleviate issues with these models, e.g., Problems 1&2

Model

A very common macro-finance setting

- All agents have log utility over consumption.
- Production is linear in capital, with *experts* more productive than *households* ($a_e > a_h$).
- Capital is freely traded at price q_t and grows evolves as

$$\frac{dK_t}{K_t} = g dt + \underbrace{\sigma dZ_t^{(1)}}_{\text{fundamental shock}}$$

- **Financial friction:** producers cannot issue equity, but can borrow/lend freely in riskless bonds at rate r_t.
    - no credit constraints
    - all results generalize to partial but limited equity issuance
- **Information structure:** extrinsic uncertainty $dZ^{(2)}$

Capital price and return

Capital price q

$$\frac{dq_t}{q_t} = \mu_{q,t} dt + \underbrace{\sigma_{q,t}^{(1)} dZ_t^{(1)}}_{\text{amplification of fundamentals}} + \underbrace{\sigma_{q,t}^{(2)} dZ_t^{(2)}}_{\text{sunspot fluctuations}}$$

Volatility of capital returns $|\sigma_R|^2$

$$\sigma_{R,t} := \sigma \binom{1}{0} + \sigma_{q,t}$$

Equilibrium

- **Price-output relation:**

$$\rho q = a_e \kappa + a_h (1 - \kappa) \quad \text{(from goods market)}$$

where κ is experts' capital share.

- **Risk-balance condition:**

$$\frac{a_e - a_h}{q} = \frac{\kappa - \eta}{\eta(1-\eta)} |\sigma_R|^2 \quad \text{(optimal portfolios when } \kappa < 1\text{)}$$

where η is experts' wealth share.

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- Risk premium:

$$\mu_q - r + \sigma\sigma_q \cdot \left(\begin{smallmatrix}1\\0\end{smallmatrix}\right) = -(\rho + g) + \left(\frac{\kappa^2}{\eta} + \frac{(1-\kappa)^2}{1-\eta}\right)|\sigma_R|^2$$

- Wealth share dynamics: $d\eta_t = \mu_{\eta,t}dt + \sigma_{\eta,t} \cdot dZ_t$ given η_0

$$\mu_\eta = \mu_\eta(\eta, \kappa, |\sigma_R|^2), \quad \sigma_\eta = (\kappa - \eta)\sigma_R$$

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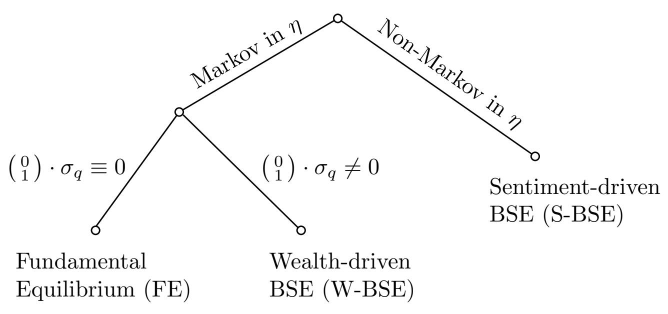
Equilibrium: Given $\eta_0 \in (0,1)$, an *equilibrium* consists of processes $(\eta_t, q_t, \kappa_t, r_t)_{t\geq 0}$ such that equations above hold for all $t \geq 0$.

Types of equilibria



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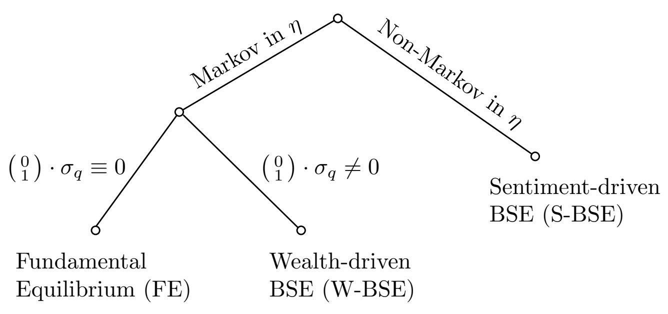
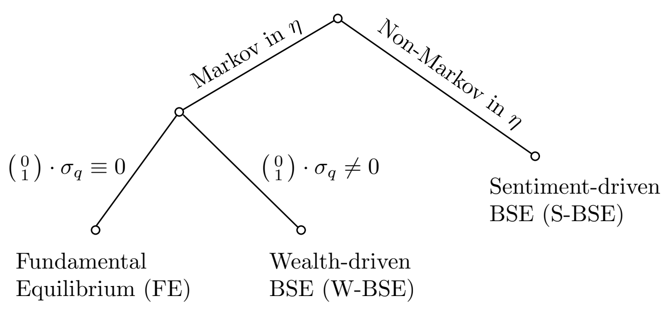


Usual solution path: imposing a Markov solution in η (i.e., $q = q(\eta)$)

- Extra conditions: dq consistent with $d\eta$ (Ito's Lemma)

$$q\sigma_q = q'\sigma_\eta, \quad q\mu_q = q'\mu_\eta + 0.5q''|\sigma_\eta|^2$$

Types of equilibria

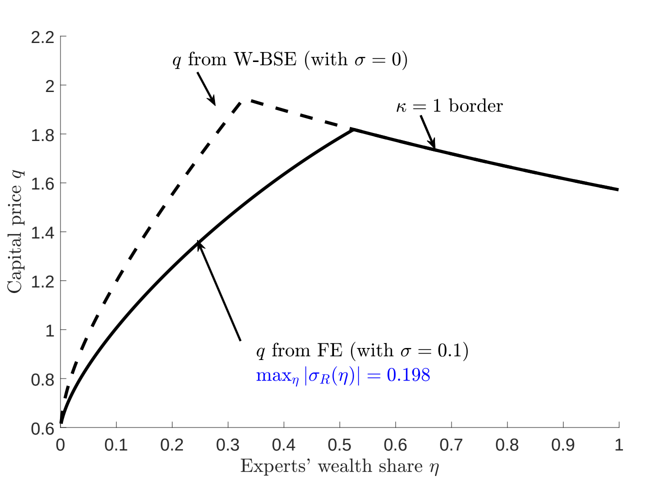


Usual solution path: imposing a Markov solution in η (i.e., $q = q(\eta)$)

- FE: widely studied (e.g., Brunnermeier and Sannikov [2016])
- W-BSE: inconsistent w/ fundamental shocks ($\sigma > 0$)
    - w/o fundamental shocks ($\sigma = 0$), there exist a W-BSE but it strongly resembles a FE with small σ.

$\Rightarrow$ No interesting new dynamics if equilibrium is Markov in η !

Fundamental equilibrium and W-BSE



Beyond wealth: sentiment-driven BSE (S-BSE)

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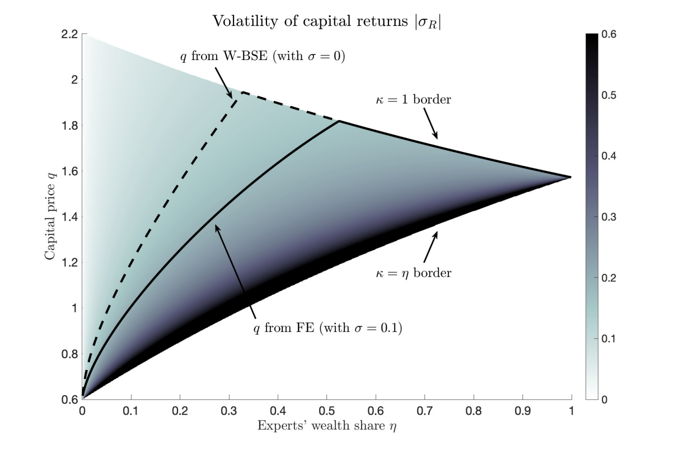
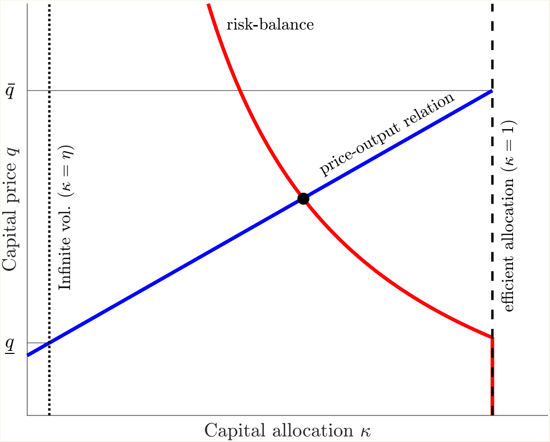
Theorem (Existence of S-BSEs):
Under mild parametric restrictions, there exists an S-BSE in which $(\eta_t, q_t)_{t\geq 0}$ remains in $\mathcal{D} := \{(\eta, q) : 0 < \eta < 1 \text{ and } \eta a_e + (1-\eta)a_h < q\bar{\rho}(\eta) \leq a_e\}$ almost-surely and possesses a non-degenerate stationary distribution.



Static indeterminacy mechanism

Price-output: $\rho q = a_e \kappa + a_h (1 - \kappa)$

Risk-balance: $\dfrac{a_e - a_h}{q} = \dfrac{\kappa - \eta}{\eta(1-\eta)} |\sigma_R|^2$



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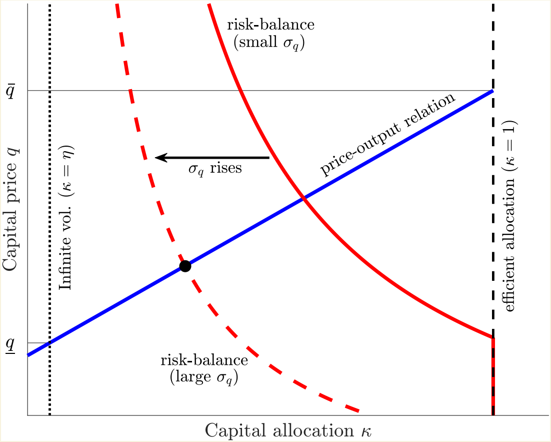


Dynamic stability mechanism

- Static indeterminacy is compatible with equilibrium only if it does not lead to violations of equilibrium conditions in the future (i.e., $(\eta_t, q_t)_{t\geq 0}$ remain in triangle $\mathcal{D}$).

- Only the risk premium is pinned down, not μ_q and r separately,

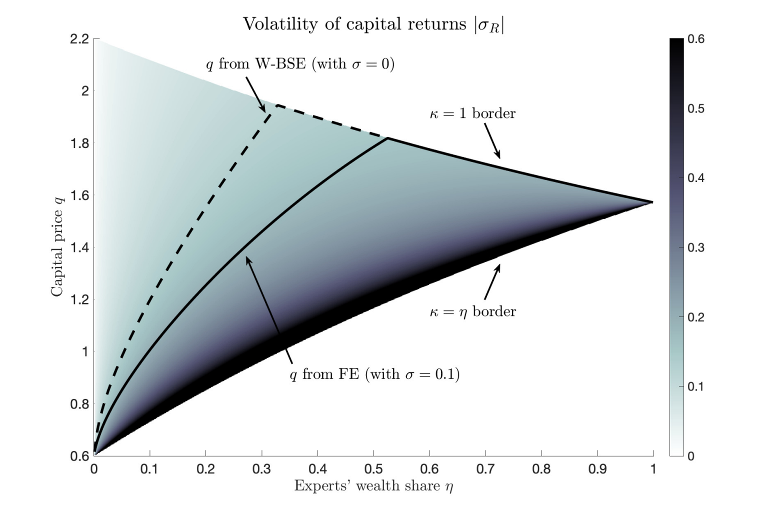
$$\mu_q - r + \sigma\sigma_q \cdot \left(\begin{smallmatrix}1\\0\end{smallmatrix}\right) = -(\rho + g) + \Big(\frac{\kappa^2}{\eta} + \frac{(1-\kappa)^2}{1-\eta}\Big)|\sigma_R|^2$$

  Hence, we use the degree of freedom to choose μ_q to ensure stochastic stability.

- Choice of μ_q is straightforward. For example, $\mu_q \to \infty$ if q falls too low, and $\mu_q \to -\infty$ if q rises too high.

- Stability requirements translate to boundary conditions.

Sentiment-driven BSE (S-BSE)



Two indeterminacies in S-BSEs

Corollary (Decoupling)

The economy can be arbitrarily coupled or decoupled from fundamentals in the following sense. Let $\gamma(\eta, q) \in [0, 1]$ be any C^1 function. An equilibrium exists such that when $\kappa < 1$, a fraction $\gamma(\eta, q)$ of return variance $|\sigma_R|^2$ is due to the fundamental shock.

Corollary (Drift indeterminacy)

The economy can feature any degree of persistence or transience in the following sense. Let $m(\eta, q)$ be any C^1 function. An equilibrium exists with $\mathbb{P}[\mu_{q,t} = m(\eta_t, q_t) \mid \kappa_t < 1]$ arbitrarily close to one. Furthermore, the inefficiency probability $\mathbb{P}[\kappa_t < 1]$ can take any value between zero and one.

Resolving puzzles with sentiment

Explicit construction with sentiment variable

- Let s_t be a pure sunspot that is irrelevant to economic fundamentals and loads on only the second shock

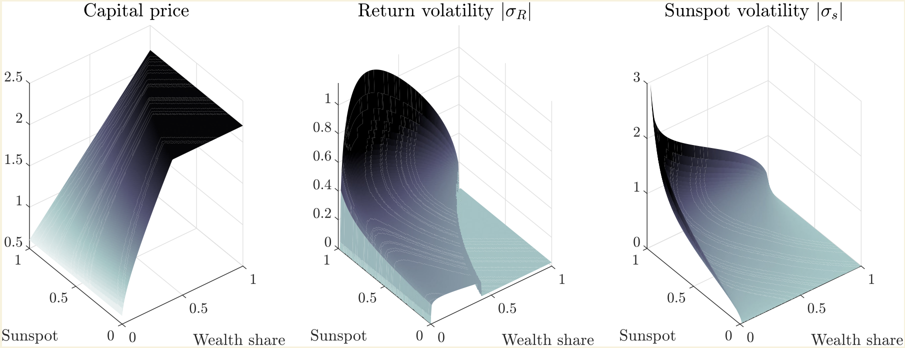
$$ds_t = \mu_{s,t} dt + \sigma_{s,t} \binom{0}{1} \cdot dZ_t, \quad s_t \in \mathcal{S}$$

- Auxiliary sunspot state variable $x_t \in \mathcal{X}$ that may only affect the drift $\mu_{s,t}$ (flexibility due to indeterminacy corollary).

Definition A *Markov S-BSE* in states $(\eta, s, x) \in (0,1) \times \mathcal{S} \times \mathcal{X}$ consists of functions $(q, \kappa, r, \sigma_\eta, \mu_\eta, \sigma_s) : (0,1) \times \mathcal{S} \mapsto \mathbb{R}$, and $\mu_s : (0,1) \times \mathcal{S} \times \mathcal{X} \mapsto \mathbb{R}$ such that the process $(\eta_t, q(\eta_t, s_t), \kappa(\eta_t, s_t), r(\eta_t, s_t))_{t \geq 0}$ is a S-BSE.

- We allow (μ_s, σ_s) to depend on η.
  - Why? It's sensible to use asset prices directly in forecasting.
  - Novel construction: fix $q(\eta, s)$, recover the σ_s process that justifies it, then set μ_s to ensure stability.

Example equilibrium construction



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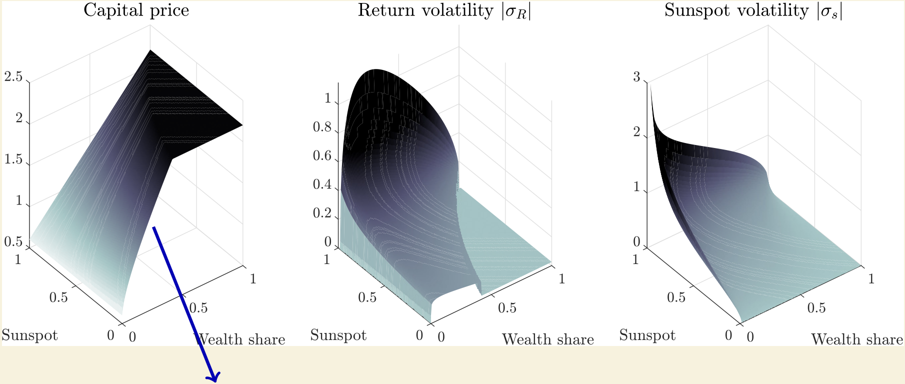
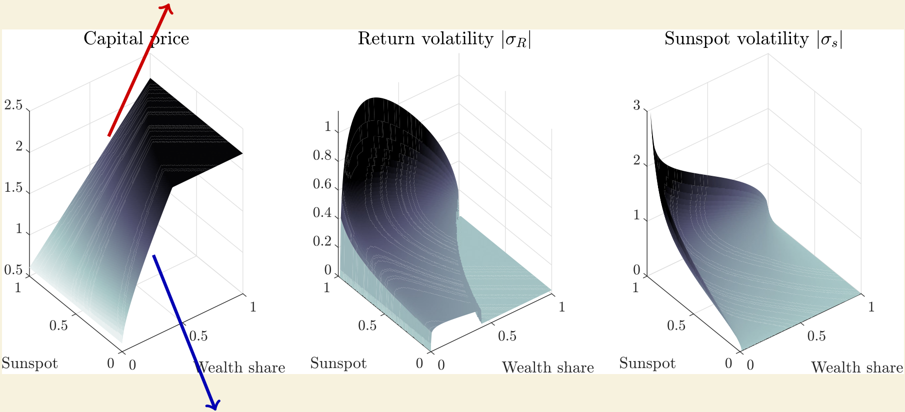


Fundamental equilibrium with $\sigma > 0$

Example equilibrium construction

Terrible equilibrium where $\kappa \approx \eta$



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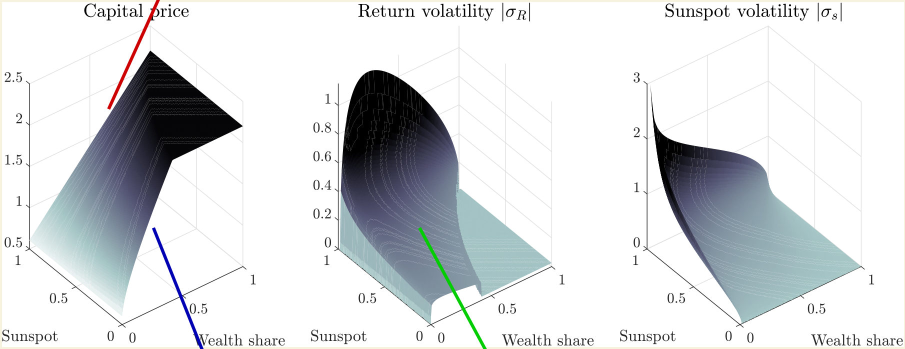
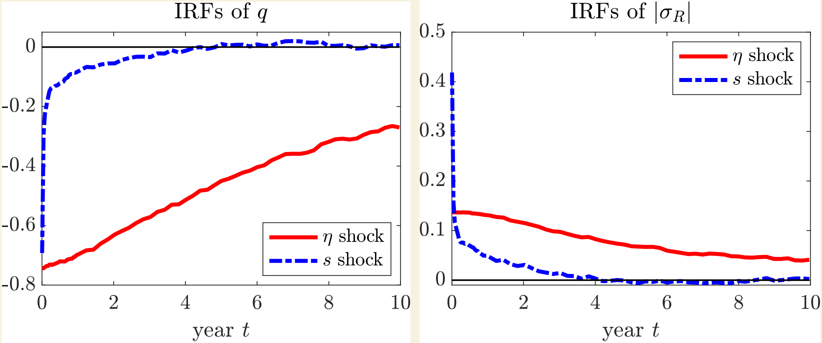


Fundamental equilibrium with $\sigma > 0$

Price-volatility relation

Fundamental vs non-fundamental busts



- The IRFs labeled "η shock" are responses to a decrease in η from $\eta_{0-} = 0.5$ to $\eta_0 = 0.2$, holding s_0 fixed at 0.1.
- The IRFs labeled "s shock" are responses to an increase in s from $s_{0-} = 0.1$ to $s_0 = 0.9$, holding η_0 fixed at 0.5.
- These shock sizes are chosen such that the initial response of q are approximately equal.

Non-fundamental crises and large amplification

Proposition (Arbitrary volatility)

Given a target variance $\Sigma^* > 0$ and under mild parameter restrictions, there exists a Markov S-BSE with stationary average return variance exceeding the target, i.e., $\mathbb{E}[|\sigma_R|^2] > \Sigma^*$.

Proposition (Volatility decoupling)

In the Markov S-BSEs constructed both the fraction of return volatility due to sentiments $|\binom{0}{1} \cdot \sigma_R|/|\sigma_R|$ and total return volatility $|\sigma_R|$ increase with s.

Booms predict crises

- Following some models of extrapolative beliefs [Barberis et al., 2015, Maxted, 2020], define an exponentially-declining weighted average of sentiment shocks:

$$x_t := x_0 + \sigma_x \int_0^t e^{-\beta_x (t-u)} dZ_u^{(2)}.$$

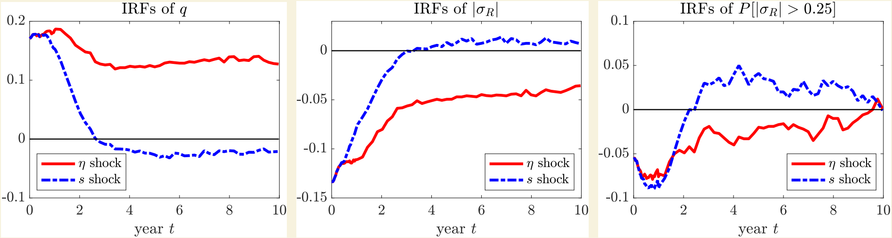
  Assume the drift of s depends on x via

$$\mu_{s,t} = b_x x_t + \hat{\mu}_s(s_t) \quad \text{with} \quad b_x \leq 0.$$

  the term $\hat{\mu}_s$ is designed to prevent non-stationarity in s_t.

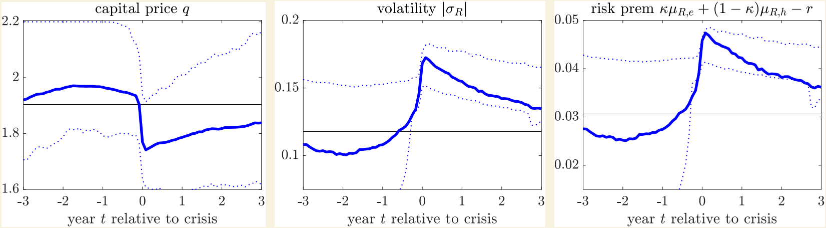
- After a series of good sentiment shocks ($dZ_t^{(2)} < 0$), s_t and x_t will be low (boom times), but this buoys $\mu_{s,t}$ and shifts conditional distributions of s_{t+h} to the right (future busts).

Booms predict crises



- The IRFs labeled "η shock" are responses to an increase in η from $\eta_{0-} = 0.5$ to $\eta_0 = 0.7$, holding s_0 fixed at 0.4.
- The IRFs labeled "s shock" are responses to a decrease in s from $s_{0-} = 0.4$ to $s_0 = 0.1$, holding η_0 fixed at 0.5.
- These shock sizes are chosen such that the initial response of q are approximately equal.

Behavior around financial crises



- Crises are defined as the bottom 3rd percentile of month-to-month log output declines.
- Conditions are improving up to 2 years before the crisis, with risk premia below average and *declining*.
- The crisis emerges suddenly and features spikes in all variables.
- These dynamics cannot be produced in the non-sunspot equilibria of the model.

Sentiment-based jumps

- Consider a broader class of solutions for the baseline model where capital price can also respond to an extrinsic jump shock, i.e.,

$$\frac{dq_t}{q_{t-}} = \mu_{q,t-}dt + \sigma_{q,t-} \cdot dZ_t - \ell_{q,t-}dJ_t,$$

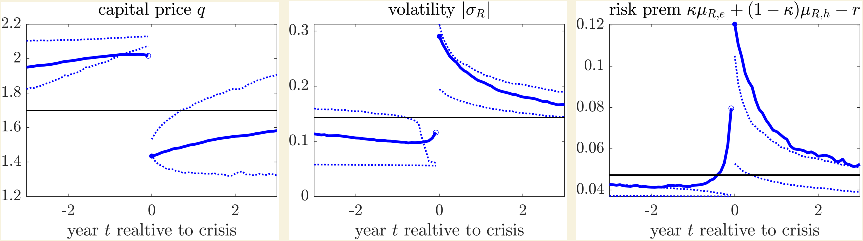
  where J is a Poisson process with intensity λ.
- The risk-balance condition

$$\frac{a_e - a_h}{q} = \frac{\kappa - \eta}{\eta(1-\eta)}\left(|\sigma_R|^2 + \frac{\lambda \ell_q^2}{\left(1 - \frac{\kappa}{\eta}\ell_q\right)\left(1 - \frac{1-\kappa}{1-\eta}\ell_q\right)}\right)$$

  disciplines overall risk but not the split between Brownian and Poisson shocks. Additional degree of freedom.
- Chosen jump sizes for exercise

$$\ell_q = \begin{cases} 0.95\ell_q^{\mathsf{max}}, & \text{if } \kappa > 0.9 \text{ and } 0.9\ell_q^{\mathsf{max}} > 0.2 \\ 0, & \text{otherwise,} \end{cases}$$

Sentiment-based jumps: behavior around crises



- Crises: bottom 3rd percentile of month-to-month log output declines.
- Crises tend to arrive after a sequence of positive fundamental shocks.
- In the years before the crisis, asset prices are high, and both volatility and risk premia are below their usual level.
- Crises arrive suddenly—with only a few months "warning" in terms of rising volatility and risk premia—and generate large movements in observables, because simulated crises often coincide with realizations of a jump.

Conclusion

- Macroeconomic models with financial frictions inherently permit sunspot volatility. These models are extremely common, so this phenomenon cannot be ignored.
- Fully-rational notion of "sentiments" can be a powerful input into macro-finance dynamics. Unbounded amplification, sharp volatility spikes, and sentiment-driven boom-bust cycles are among the possibilities.
- Our results suggest a modicum of caution. Numerical techniques used to solve DSGE models with financial frictions implicitly select an equilibrium, without any explicit justification. A deeper analysis of refinements still remains to be done.
- **Policy?**
  - Deposit insurance less effective, because run-like behavior can be an asset-side phenomenon.
  - Capital requirements, bailouts, etc, are likely less effective when volatility is decoupled from balance sheets.